DEPARTMENT OF MATHEMATICS
MODEL PAPERS

CBCS/ SEMESTER SYSTEM

(W.e.f 2020-21 Admitted Batch)

B.A./B.Sc. MATHEMATICS

COURSE-I, DIFFERENTIAL EQUATIONS

MATHEMATICS MODEL PAPER

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks5 X 5 M=25 M

1. Solve $(1 + e^{x/y})\,dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0.$
2. Solve $(y - e^{\sin^{-1}x})\frac{dx}{dy} + \sqrt{1 - x^2} = 0$
3. Solve $y + px = p^2x^4.$
4. Solve $(px - y)(py + x) = 2p$
5. Solve $(D^2 - 3D + 2) = \cosh x$
6. Solve$(D^2 - 4D + 3)y = \sin 3x \cos 2x.$
7. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}\sin 2x.$
8. Solve $x^2y'' - 2x(1 + x)y' + 2(1 + x)y = x^3$

SECTION - B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9 a) Solve $x\frac{dy}{dx} + y = y^2 \log x.$

(Or)

9 b) Solve $(y + \frac{1}{3}y^3 + \frac{1}{2}x^2)\,dx + \frac{1}{4}(x + xy^2)\,dy = 0.$

10 a) Solve$p^2 + 2py\cot x = y^2.$

(Or)

10 b) Find the orthogonal trajectories of the family of curves

$x^{2/3} + y^{2/3} = a^{2/3}$ where ‘a’ is the parameter.

11 a) Solve$(D^3 + D^2 - D - 1)y = \cos 2x$.

(Or)

11 b) Solve$(D^2 - 3D + 2)y = \sin e^{-x}$.

12 a) Solve $(D^2 - 2D + 4)y = 8(x^2 + e^{2x} + \sin 2x)$

(Or)

12 b) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x \sin x$

13 a) Solve $(D^2 - 2D)y = e^x \sin x$ by the method of variation of parameters.

(Or)

13 b) Solve $3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$

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(w.e.f. 2020-21 Admitted Batch)

B.A./B.Sc. MATHEMATICS

COURSE-II, THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY

Time: 3Hrs **Max.Marks:75 M**

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M

1. Find the equation of the plane through the point (-1,3,2) and perpendicular to the planes x+2y+2z=5 and 3x+3y+2z=8.
2. Find the bisecting plane of the acute angle between the planes 3x-2y-6z+2=0, -2x+y-2z-2=0.
3. Find the image of the point (2,-1,3) in the plane 3x-2y+z =9.
4. Show that the lines $2x + \ -4 = 0 = y + 2z$ and $x + 3z - 4 = 0$, $2x + 5z - 8 = 0$ are coplanar.
5. A variable plane passes through a fixed point (a, b, c). It meets the axes in A,B,C. Show that the centre of the sphere OABC lies on $ax^{-1}+by^{-1}+cz^{-1}=2$.
6. Show that the plane 2x-2y+z+12=0 touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.
7. Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$
8. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with its vertex at (1, 1, 1).

SECTION - B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9(a) A plane meets the coordinate axes in A, B, C. If the centroid of ΔABC is (a,b,c), show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3.$

(OR)

(b) A variable plane is at a constant distance p from the origin and meets the axes in A,B,C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$.

10(a) Find the shortest distance between the lines

$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$; $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$.

(OR)

(b) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.

11 (a) Show that the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z=2$; $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lie on the same sphere and find its equation.

(OR)

(b) Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at (1,-2,1) and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$.

12 (a) Find the limiting points of the coaxial system of spheres $x^2+y^2+z^2-8x+2y-2z+32=0$, $x^2+y^2+z^2-7x+z+23=0$.

(OR)

(b) Find the equation to the cone with vertex is the origin and whose base curve is $x^2+y^2+z^2+2ux+d=0$.

13 (a) Prove that the equation $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$ represents a cone that touches the coordinate planes and find its reciprocal cone.

(OR)

(b) Find the equation of the sphere $x^2+y^2+z^{2-}2x+4y-1=0$ having its generators parallel to the line $x=y=z$.

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B.A./B.Sc. MATHEMATICS

COURSE-III, ABSTRACT ALGEBRA

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M

1. Show that the set G=$\{x/x = 2^a 3^b \text{ and } a, b \in Z\}$ is a group under multiplication

2. Define order of an element. In a group G, prove that if $a \in G$ then $O(a) = O(a)^{-1}$.

3. If H and K are two subgroups of a group G, then prove that HK is a subgroup ⇔ HK=KH

4.If G is a group and H is a subgroup of index 2 in G then prove that H is a normal subgroup.

5. Examine whether the following permutations are even or odd

i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 4 & 3 & 2 & 5 & 7 & 8 & 9 \end{pmatrix}$

ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$

6. Prove thata group of prime order is cyclic.

7.Prove that the characteristic of an integral domain is either prime or zero.

8. If F is a field then prove that $\{0\}$ and F are the only ideals of F.

SECTION - B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9 a) Show that the set of n^{th} roots of unity forms an abelian group under multiplication.

(Or)

9 b) In a group G, for $a, b \in G$, O(a)=5, b ≠ e and $aba^{-1} = b^2$. Find O(b).

10 a) The Union of two subgroups is also a subgroup ⇔ one is contained in the other.

(Or)

b) State and prove Langrage's theorem.

11 a) Prove that a subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

(Or)

11 b) State and prove fundamental theorem of homomorphisms of groups.

12 a) Let S_n be the symmetric group on n symbols and let A_n be the group of even permutations. Then show that A_n is normal in S_n and $O(A_n) = \frac{1}{2}(n!)$

(Or)

12 b)prove thatevery subgroup of cyclic group is cyclic.

13 a) Prove that every finite integral domain is a field.

(Or)

13 b) Define principal idea. Prove that every ideal of Z is a principal ideal.

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B.A./B.Sc. MATHEMATICS

COURSE-IV, REAL ANALYSIS

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M

1.Prove that every convergent sequence is bounded.

2. Show that $\lim\left(\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\text{-----------------}+\frac{1}{(n+n)^2}\right)=0$.

3. Test the convergenceof the series $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1}-n\right)$.

4. Examine for continuity of the function *f* defined by $f(x)=|x|+|x-1|$ at x=0 and 1.

5. Show that $f(x)=x\sin\frac{1}{x},\ x\neq 0; f(x)=0, x=0$ is continuous but not derivable at x=0.

6.Verify Rolle's theorem for the function f(x) = x^3- $6x^2$+11x-6 on $[1,3]$.

7. If $f(x)=x^2\ \forall\, x\in[0,1]$ and $p=\{0,\frac{1}{4},\frac{2}{4},\frac{3}{4},1\}$ then find L(p, f) and U(p, f).

8.prove that if $f:[a,b]\to R$ is continuous on [a, b] then f is R- integrable on [a, b].

SECTION –B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9.(a)If $s_n=1+\frac{1}{2!}+\frac{1}{3!}+------+\frac{1}{n!}$ then show that $\{s_n\}$ converges.

(OR)

(b) State and prove Cauchy's general principle of convergence.

10.(a) State and Prove Cauchy's nth root test.

(OR)

(b) Test the convergence of $\sum \frac{x^n}{x^n+a^n}$ $(x > 0, a > 0)$.

11.(a) Let f: R→ R be such that

$$f(x) = \frac{\sin(a+1)x+\sin x}{x} \quad \text{for } x < 0$$

$$= c \qquad \text{for } x = 0$$

$$= \frac{(x+bx^2)^{1/2}-x^{1/2}}{bx^{3/2}} \quad \text{for } x > 0$$

Determine the values of a, b, c for which the function f is continuous at x=0.

(OR)

(b) Define uniform continuity, If a function f is continuous on [a b] then f is uniformly continuous on [a b]

12.(a) Using Lagrange's theorem, show that $x > \log(1+x) > \frac{x}{(1+x)} \forall x > 0$.

(OR)

(b) State and prove Cauchy's mean value theorem.

13.(a) State and prove Riemman's necessary and sufficient condition for R- integrability.

(OR)

(b) Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}$.

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B.A./B.Sc. MATHEMATICS

COURSE-V, LINEAR ALGEBRA

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M

1. Let p, q, r be fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F, such that px+qy+rz=0 is a vector subspace of $V_3(R)$.

2. Define linearly independent &linearly dependent vectors in a vector space. If α, β, γ are linearly independent vectors of $V(R)$ then show that $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ are also linearly independent.

3. Prove that every set of $(n+1)$ or more vectors in an n dimensional vector space is linearly dependent.

4. The mapping T : $V_3(R) \rightarrow V_3(R)$ is defined by T(x,y,z) = (x-y,x-z). Show that T is a linear transformation.

5. Let $T: R^3 \rightarrow R^2$ and $H: R^3 \rightarrow R^2$ be defined by T (x, y, z)= (3x, y+z) and H (x, y, z)= (2x-z, y). Compute i) T+H ii) 4T-5H iii) TH iv) HT.

6. If the matrix A is non-singular, show that the eigen values of A^{-1} are the reciprocals of the eigen values of A.

7. State and prove parallelogram law in an inner product space V(F).

8. Prove that the set S = $\left\{\left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right)\right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

SECTION - B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9(a)) Define vector space. Let V (F) be a vector space. Let W be a non empty sub set of V. Prove that the necessary and sufficient condition for W to be a subspace of V is $a, b \in F$ and $\alpha, \beta \epsilon V => a\alpha + b\beta \epsilon W$.

(OR)

(b) Prove that the four vectors (1,0,0), (0,1,0), (0,0,1) and (1,1,1) of $V_3(C)$ form linearly dependent set, but any three of them are linearly independent.

10(a)Define dimension of a finite dimensional vector space. If W is a subspace of a finite dimensional vector space V(F) then prove that W is finite dimensional and dim W$\leq n$.

(OR)

(b) If W be a subspace of a finite dimensional vector space V(F) then Prove that $\dim {}^{V}/_{W} = \dim V - \dim W$.

11(a) Find T (x, y, z) where $T: R^3 \rightarrow R$ is defined by T (1, 1, 1) =3, T (0, 1, -2) =1, T (0, 0, 1) = -2

(OR)

(b) State and prove Rank Nullity theorem.

12(a) Find the eigen values and the corresponding eigen vectors of the matrix

$$A=\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}.$$

(OR)

(b) State and prove Cayley-Hamilton theorem.

13(a) State and prove Schwarz's inequality in an Inner product space V(F).

(OR)

(b) Given $\{(2,1,3),(1,2,3),(1,1,1)\}$ is a basis of R^3(R).Construct an orthonormal basis using Gram-Schmidt orthogonalisation process.

MODEL TEST PAPER - 1

B.A./B.Sc. DEGREE EXAMINATION

THIRD YEAR - SEMESTER-6

MATHEMATICS PAPER - 7A

Time : 3 Hours LAPLACE TRANSFORMS Max. Marks : 75

SECTION-A (5 × 5 = 25 Marks)

Answer any *FIVE* of the following questions.

1. Find $L\{7e^{2t} + 9e^{-2t} + 5\cos t + 7t^3 + 5\sin 3t + 2\}$.

2. Find $L\{\cosh at \cos at\}$

3. Find $L\{te^{3t} \sin 2t\}$.

4. Evaluate $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t} dt$.

5. Find $L^{-1}\left(\frac{p^2}{p^4 - a^4}\right)$.

6. Find $L^{-1}\left[\frac{3p+1}{p^2 - 2p - 3}\right]$.

7. Find $L^{-1}\left\{Tan^{-1}\left(\frac{a}{p}\right) + Cot^{-1}\left(\frac{p}{b}\right)\right\}$.

8. Find $L^{-}\left\{\frac{1}{p}\log\left(\frac{p+2}{p+1}\right)\right\}$

SECTION- B (5 × 10 = 50 Marks)

Answer *ALL* of the following questions

9. a) If a function $F(t)$ is piecewise continuous on every finite interval in the range $t \geq 0$ and satisfies $|F(t)| < Me^{at}\ \forall\ t \geq t_0$ and for some constants and M, then prove that the Laplace transform of $F(t)$ exists for all $p > a$.

Or

b) Find the Laplace transform of $(\sin t - \cos t)^3$.

10. a) State and prove initial value theorem.

Or

b) i) Find $L\{e^{-t}\cos^2 t\}$

ii) If $L[F(t)] = \frac{9p^2 - 12p + 15}{(p-1)^3}$, find $L[F(3t)]$ using change of scale property

11. a) Evaluate $L\left\{\frac{1 - \cos at}{t}\right\}$

OR

b) Using Laplace transform, evaluate $\int_{0}^{\infty} \frac{\cos at - \cos bt}{t} dt$.

12. a) Find $L^{-1}\left\{\frac{3}{p^2-3}+\frac{3p+2}{p^3}-\frac{3p-27}{p^2+9}+\frac{6-30\sqrt{p}}{p^4}\right\}$.

Or

b) Find $L^{-1}\left\{\frac{e^{-\pi p}(p+1)}{p^2+p+1}\right\}$.

13. a) Find the Inverse Laplace Transformation of $\frac{p+3}{(p^2+6p+13)^2}$.

Or

b) State and prove convolution theorem.